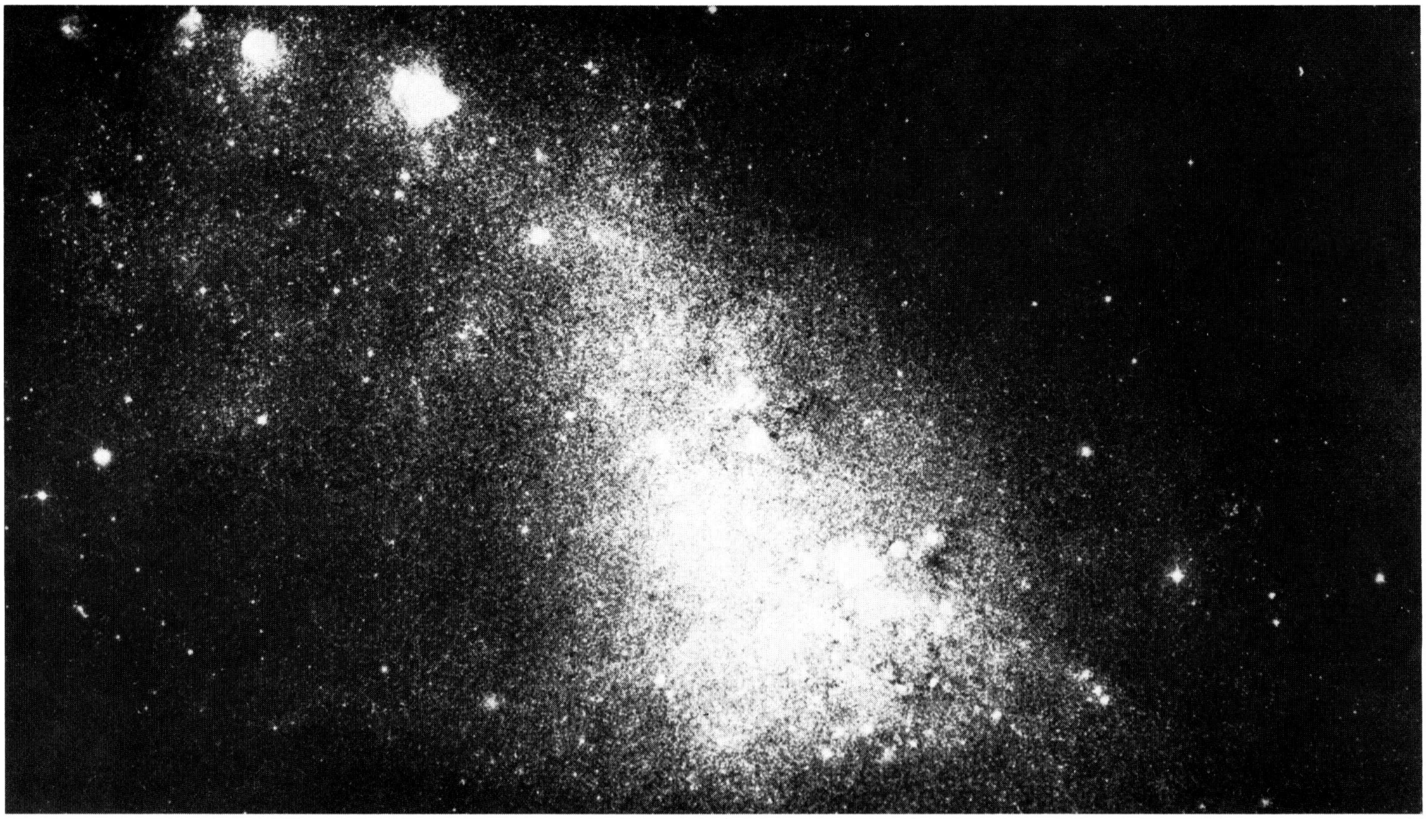

The Small Magellanic Cloud is the second-nearest galaxy to our Milky Way, close enough for its stars to be studied as individuals. This photograph is a 60-minute exposure in blue light, taken with the 1-meter Schmidt telescope of the European Southern Observatory in Chile. North is up and the height of this picture is about 2°. Copyright ESO.

Laboratory Exercises in Astronomy — Cepheid Variables and the Cosmic Distance Scale

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THE MOST ACCURATE means of determining the distances to the nearest stars is known as the method of *trigonometric parallax.* This involves measuring the angular shift of a star with regard to much more remote objects, as seen from different positions in the earth's orbit. The distance of a star in parsecs (1 parsec = 3.26 light-years) is equal to the reciprocal of its parallax in seconds of arc.

Every parallax determination has a probable error of about 0.005 second, as a rule. Hence, as astronomers try to determine the distances of increasingly remote stars, the trigonometric parallax method gives less and less accurate results. At a distance of 200 parsecs, the probable error is about as large as the parallax itself. Despite the success of modern astronomers in decreasing the errors of parallax determinations, this method of determining star distances is reliable only for objects in the sun's own "backyard."

Other methods are needed for objects more distant than a few hundred parsecs. In this exercise we will use *Cepheid variable stars,* those that change in brightness with periods from 1 to 100 days in the manner of the 5.4-day variation of δ Cephei. Some of these supergiant stars are more than 10,000 times as luminous as our sun and thus can be seen at great distances, being recognizable in the nearer galaxies. The Cepheids played a key role, in the early years of this century, in providing proof that there are other galaxies than our own. Although today we readily acknowledge that other galaxies exist besides the Milky Way, it wasn't until the 1920's that this idea became commonly accepted.

CEPHEID VARIABLES

The first known Cepheid was δ Cephei, discovered in 1784 by an English amateur astronomer, John Goodricke. About 1879, A. Ritter theorized that the light variations of these stars are due to pulsations, the star alternately expanding and contracting. Later astronomers verified this idea by spectroscopic observations. In this exercise, we are not concerned with why the stars pulsate but rather how they are used as distance indicators.

At the beginning of this century the distance of the Small Magellanic Cloud (today recognized as a neighbor galaxy) was unknown. On Harvard Observatory photographs of it, Henrietta S. Leavitt had discovered many faint Cepheids. In 1912 she showed, in a detailed study of about two dozen of these Cepheids, that there was a clear-cut correlation between the apparent magnitudes and the periods of these stars, in the sense that the longer-period stars are the brighter.

Since all the stars in the Small Magellanic Cloud are basically at the same distance from the sun, it follows that the apparently brighter Cepheids in it are in fact intrinsically more luminous. In other words, the period of a Cepheid is an indicator of its intrinsic luminosity, and in the following problem we shall use this property to determine the distance of the Small Magellanic Cloud.

STEP 1

Shown on the page that follows are light curves of four Cepheid variables in the Small Magellanic Cloud, based on photographic observations by H. C. Arp in yellow light. For each star, read off the apparent magnitudes at maximum and at

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minimum to the nearest 0.1 magnitude; then take the average of these two values. Also, find for each star its period in days, from the interval between successive maxima. Take the logarithm of the period, to two decimal places.

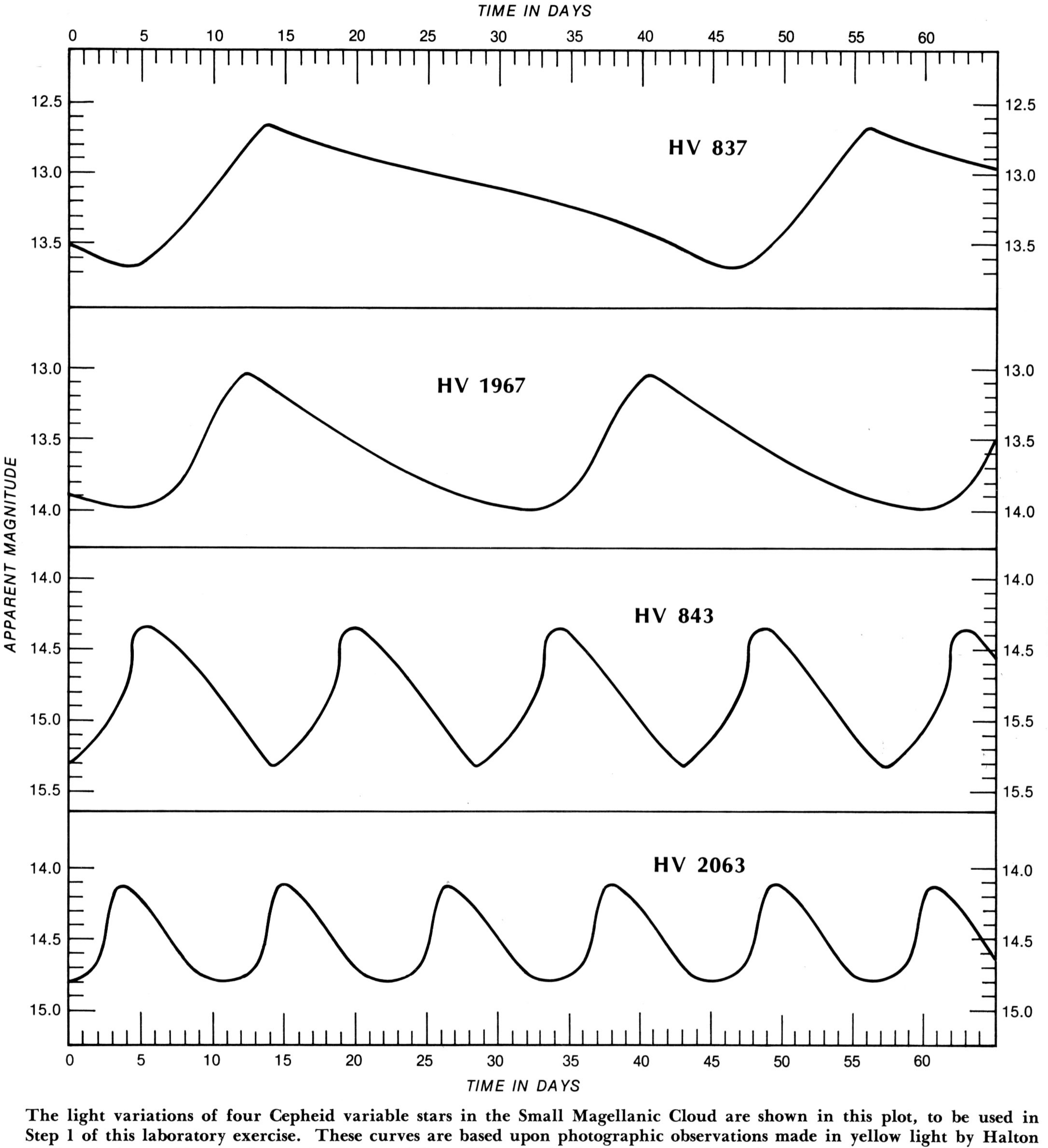
On the graph on the facing page, plot for each star the mean apparent magnitude as the ordinate against the logarithm of the period. To increase the number of points, plot the data from Table I, which are also for Small Cloud Cepheids observed in yellow light by Arp. Next, draw a straight line to fit the data points as well as possible.

TABLE I

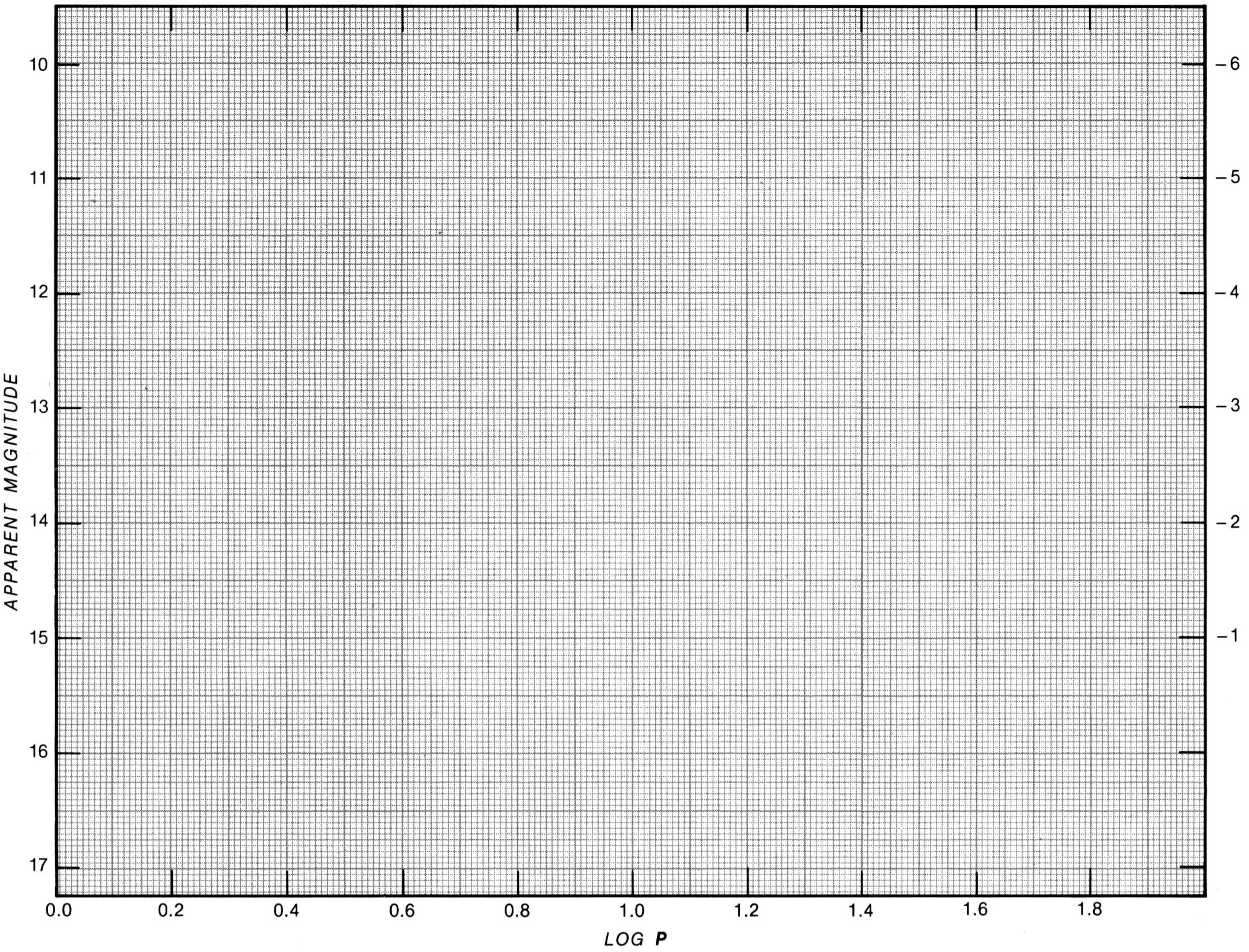
Cepheids in the Small Cloud

HV	*log P*	m_v	*HV*	*log P*	m_v
2019	0.21	16.8	2060	1.01	14.3
2035	0.30	16.7	1873	1.11	14.7
844	0.35	16.3	1954	1.22	13.8
2046	0.41	16.0	847	1.44	13.8
1809	0.45	16.1	840	1.52	13.4
1987	0.50	16.0	11182	1.60	13.6
1825	0.63	15.6	1837	1.63	13.1
1903	0.71	15.6	1877	1.70	13.1
1945	0.81	15.2			

This plot gives us the relation between apparent magnitude and period for the Cepheids in that galaxy. Since all the stars in the Small Cloud are at essentially the same distance from us, the plot can also be regarded as a *period-luminosity* (P-L) *relation* which is as yet uncalibrated. We now proceed to calibrate it, as described on the next page.



The light variations of four Cepheid variable stars in the Small Magellanic Cloud are shown in this plot, to be used in Step 1 of this laboratory exercise. These curves are based upon photographic observations made in yellow light by Halton C. Arp, using a 46-cm refractor in South Africa. His magnitude scale was calibrated photoelectrically and so is very reliable. The letters HV indicate a variable star discovered by Harvard Observatory astronomers. The great majority of the variables in the Small Cloud were found on photographs taken with Harvard telescopes in Peru and in South Africa.



This grid or its equivalent is needed for plotting in this exercise. Note that the apparent-magnitude scale at left and the absolute-magnitude scale at right are quite independent.

SHAPLEY'S CALIBRATION

If two stars have the same intrinsic luminosity, their apparent brightnesses are inversely proportional to the squares of their distances. This straightforward fact can be restated as a formula connecting the apparent magnitude m of a star and its distance d in parsecs with the same star's absolute magnitude M (defined as the magnitude it would have if seen from a standard distance of 10 parsecs).

This formula is

$$M = m + 5 - 5 \log d. \qquad (1)$$

Thus, if the absolute and apparent magnitudes of any star are known, we can calculate its distance from this formula. Similarly, if the P-L relation can be calibrated in terms of absolute magnitude (instead of apparent magnitude as used by Miss Leavitt in 1912), then knowledge of a Cepheid's period would yield its distance, since the only other quantity needed is the apparent magnitude and this is easily observed.

In 1918 Harlow Shapley provided a calibration, later slightly revised by him, which became widely accepted by astronomers. Table II shows Shapley's P-L relation, with absolute visual magnitudes for some values of the logarithm of the period.

It should be emphasized that any such calibration is very difficult to make. Even the nearest Cepheid in our galaxy is too remote for its distance to be determined by the trigonometric parallax method. However, statistical information about the distances of the brighter Cepheids can be derived from their observed motions. Most of the Cepheids used by Shapley were in globular clusters in the Milky Way.

STEP 2

Plot Shapley's data on the same graph as Arp's, but using the right-hand scale on the ordinate (y) axis, this time in terms of absolute magnitude M. Draw a straight line to fit the points. This is the calibrated P-L relation that for many years was used in determining the distances to objects containing Cepheids. The line through Shapley's data should be nearly parallel to that through Arp's.

TABLE II

Shapley's Period-Luminosity Curve

log P	M_V	*log P*	M_V	*log P*	M_V
0.0	−0.4	+0.8	−2.2	+1.4	−4.4
+0.2	−0.8	+1.0	−2.9	+1.6	−5.1
+0.4	−1.2	+1.2	−3.6	+1.8	−5.8
+0.6	−1.6				

Determine the vertical difference $m - M$ between the two curves at several places (to reduce the effect of any small difference in slope) and take the average. From this difference $m - M$, known as the *distance modulus*, use equation (1) to calculate the distance to the Small Magellanic Cloud.

BAADE'S CALIBRATION

In 1923, Edwin Hubble at Mount Wilson Observatory was able to find 12 Cepheids in the Andromeda nebula (M31) and 22 in the great Triangulum nebula (M33). By using essentially the same method that you have just applied, Hubble was able to announce the distance of M31 as about 285,000 parsecs. So great a distance made it clear that the Andromeda nebula and similar systems were great aggregations of stars, comparable to our Milky Way in their own right.

A few years later, astronomers realized that interstellar dust in our Milky Way dimmed galaxies other than our own,

The Andromeda galaxy M31 flanked by its satellites, M32 (lower right) and NGC 205 (above). Photograph from Space Division, Chrysler Corporation.

making them appear too far away. Because of this the distance of M31 was revised downward, and a figure of 230,000 parsecs was generally accepted. In addition, distances to other galaxies were determined by Shapley's calibrated P-L relation. Thus, the distance scale of the universe was based on Shapley's work.

But as we shall now see, these distances were seriously underestimated. A major revision was made in 1952 by Walter Baade, from the first photographs of the Andromeda galaxy taken with the new 200-inch Palomar telescope. Baade had previously discovered that stars basically can be divided into two age categories. *Population I* stars are relatively young, hot ones, typically found in the spiral arms of galaxies, while *Population II* stars are old objects, characteristically found in globular clusters and in the halos of galaxies. In his 1952 photographs of M31 with the 200-inch telescope, Baade found that only the brightest of the Population II stars could be actually photographed, although fainter ones were expected to show up. This implied that the Andromeda galaxy must be even more distant than previously believed.

Since he could find no Cepheids in the globular clusters of M31 but many in the spiral arms, he deduced that the Cepheids in the globulars must be Population II and that the spiral-arm Cepheids are Population I. Along with this came the realization that the globular-cluster Cepheids, used by Shapley in his P-L relation calibration, must be about 1.5 magnitudes fainter than the Cepheids observed by Miss Leavitt in the Magellanic Clouds, which are of Population I. This discovery led to a revision of the extragalactic distance scale by a factor of slightly more than two. Although it was popularly written that the universe had become "twice as big," of course it was only the numerical values of the distances that changed.

STEP 3

Calculate the revised distance to the Small Magellanic Cloud which results if the distance modulus is changed by − 1.5 magnitudes.

AN ALTERNATIVE CALIBRATION

Even after Baade's contribution, additional revision may be needed to the P-L relation. The fundamental problem remains, as in Shapley's day, determining the zero point of the P-L relation. There is still uncertainty about the accuracy of the statistical methods Shapley used to infer Cepheids' distances from their motions. Therefore, modern astronomers have sought other observational means of establishing the zero point.

In 1961, Robert Kraft at Lick Observatory deduced the absolute magnitudes of six Population I Cepheids that were members of open clusters, which are also of Population I and whose distances can be found in other ways. By themselves, these six stars are insufficient to define completely the zero point and the slope of the curve. But, by determining the intrinsic colors of these six stars, Kraft was able to extend his list to include 26 other Cepheids not associated with any cluster. A selection of the 32 Cepheids is given in Table III with an asterisk affixed to the six in open clusters.

TABLE III

Kraft's Period Luminosity Curve

Star	*log P*	M_V	*Star*	*log P*	M_V
SU Cas	0.29	− 1.7	*U Sgr	0.83	− 3.5
*EV Sct	0.49	− 2.4	Eta Aql	0.86	− 3.5
SS Sct	0.56	− 2.4	RX Cam	0.90	− 3.7
SU Cyg	0.58	− 2.8	*DL Cas	0.90	− 3.7
Y Lac	0.64	− 2.8	*S Nor	0.99	− 3.7
FF Aql	0.65	− 3.1	Z Lac	1.04	− 4.1
*CF Cas	0.69	− 3.4	RW Cas	1.17	− 4.5
V350 Sgr	0.71	− 3.0	Y Oph	1.23	− 5.3
*CV Mon	0.73	− 3.0	T Mon	1.34	− 5.6
RR Lac	0.81	− 3.4	SV Vul	1.65	− 6.4

STEP 4

On the same graph that contains Arp's and Shapley's work, also plot the values of *M* and log *P* from Table III. Then draw the straight line that gives the best fit to these points. In the same way as before, determine the distance modulus *m* − *M* for the Small Magellanic Cloud, and calculate the distance from (1).

FUTURE DEVELOPMENTS

Although the Cepheid variables continue to be powerful indicators of the cosmic distance scale, astronomers now tend to believe that the Cepheids in one galaxy may not be physically identical with those of the same period in another galaxy. The slope of the P-L relation may be different from one galaxy to the next, depending upon the relative abundances of the elements in each. Moreover, within any one galaxy, the slopes of the P-L relations for the two populations of stars may not be the same.

Therefore, even though the distance scale is more accurately known today than it was during Shapley's time, we must await additional observations and theoretical models of Cepheid variables before astronomers can estimate just how accurate it really is.

NOTES FOR TEACHERS

Reviews of the P-L relationship for Cepheids can be found in an article by Walter Baade (*Publications* of the Astronomical Society of the Pacific, *68*, 5, 1956); O. Struve and V. Zebergs, *Astronomy of the 20th Century* (Macmillan, 1962); R. Berendzen, R. Hart, and D. Seeley, *Man Discovers the Galaxies* (Neale Watson, 1976); and J. Pasachoff, *Contemporary Astronomy* (Saunders, 1977).

The data by H. Arp are cited from *Astronomical Journal, 65,* 404, 1960; those by R. P. Kraft are from *Astrophysical Journal, 133,* 39, 1961.

To include more historical flavor in the exercise, you might consider having the students plot H. Shapley's original data (*Astrophysical Journal, 48,* 107, 1918).

The best current estimates for the distance to the Small Magellanic Cloud are between 53,000 and 60,000 parsecs. G. de Vaucouleurs (*Astrophysical Journal, 223,* 730, 1978) got 53,000 parsecs, using five different kinds of distance indicators — novae, Cepheids, RR Lyrae variables and horizontal branch stars, supergiants, and eclipsing binaries. Recent work by S. van den Bergh agrees with his result. A. Sandage and G. Tammann (*Astrophysical Journal, 167,* 293, 1971) deduced a distance which after adjustment by de Vaucouleurs is still 15 percent larger. The values differ chiefly because of different treatment of interstellar reddening.