Laboratory Exercises in Astronomy — Hubble's Law

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Cosmology is that branch of astronomy which deals with the structure and evolution of the universe as a whole. It is a remarkable fact that a vital clue to the nature of the universe is revealed by a very simple observation: the sky becomes dark after the sun sets. In a stationary universe of infinite extent, uniformly strewn with stars, our line of sight would always end at the surface of a star, and the whole sky should therefore appear bright like the sun. Why, then, is the sky dark at night? This contradiction is known as Olbers' paradox. It is resolved nowadays as being due to the expansion of the universe, distant sources receding from us at speeds so high that the intensity of light received from them is greatly reduced.

Thus, from this simplest of astronomical observations can be deduced the expansion of the universe, a phenomenon which manifests itself in the motion of galaxies away from the observer, no matter where in the universe he is situated.

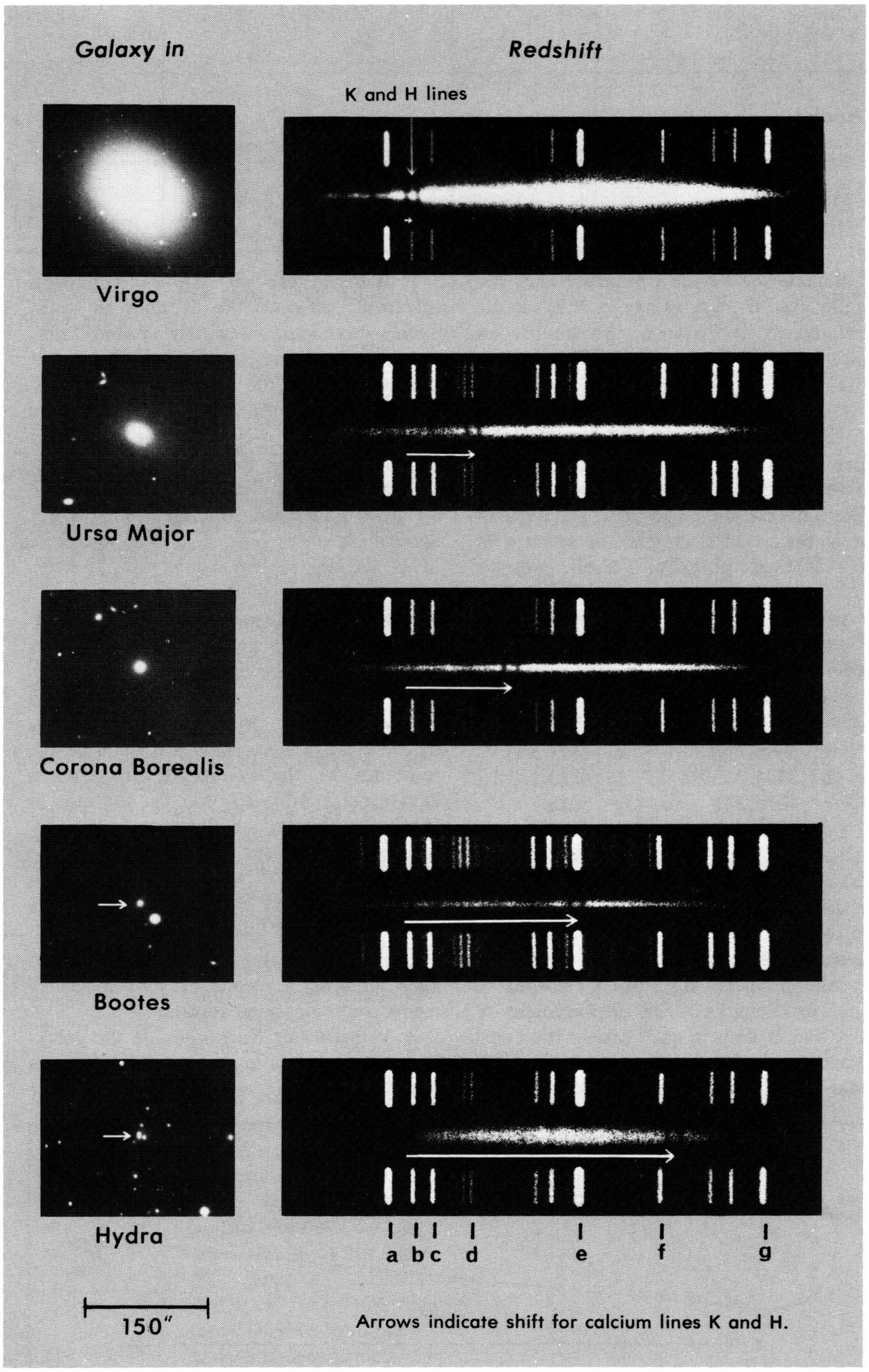
Although this general recession of the galaxies had been known since about 1920 from the observations of V. M. Slipher and others, the discovery of the expansion of the universe is invariably associated with the name of Edwin Hubble. In 1929, he was able to show that the galaxies seem to be receding with velocities that are proportional to their distances from us. Hubble's law can be written as

$$V = H \times D,$$

where V is the recessional velocity in kilometers per second, and D is the distance of the galaxy in megaparsecs (one Mpc. equals one million parsecs or 3.3 million light-years). H is called Hubble's constant and is expressed in kilometers per second per megaparsec; it is a measure of how rapidly the cosmic expansion is proceeding at the present time.

In his pioneering studies, Hubble found that H was about 540 — that is, a galaxy one megaparsec distant would be receding at 540 kilometers per second; a galaxy at two megaparsecs would recede at 1,080 kilometers per second, and so on. Although Hubble's law is now firmly established, his numerical value for H was much too large, because he underestimated the distances of galaxies. A currently accepted value, reported by Allan Sandage and G. A. Tammann in 1976, is about 50 kilometers per second per megaparsec, less than a tenth of Hubble's original estimate.

The aim of this laboratory exercise, which is based on one used at the University of Keele in England, is to verify Hubble's law and to determine the Hubble constant. For this purpose, we need a sample of galaxies for which we shall determine recessional velocities and distances. Such a sample is conveniently provided by the accompanying Hale Observatories photographs of five galaxies and their spectra. All five objects are members of clusters of galaxies. The recessional velocity of each galaxy is found by measuring the displacements of spectral lines toward the red end of the spectrum. We shall determine the distances by using the fact that galaxies of the type shown all have approximately the same linear diameter, which we shall take here to be 0.03 megaparsec (about 100,000 light-years),



Images and spectra of five galaxies are in this diagram adapted from Hale Observatories photographs. Wavelengths of comparison lines are in the text.

VELOCITY DETERMINATIONS

Galaxy in	Distance on spectrum from comparison line (millimeters)		Wavelength difference (angstroms)		Redshifted wavelength (angstroms)		Recession velocity (km./sec.)	
	K	H	K	H	K	H	K	H
Virgo								
Ursa Major								
Corona Borealis								
Bootes								
Hydra								

the diameter of our own galaxy. Thus the angular size of each galaxy in this sample is related to its distance: the smaller an object appears, the greater its distance.

PROCEDURE

A millimeter rule and a sheet of graph paper are needed for this exercise, and a simple pocket calculator is desirable.

Reproduced on page 299 at approximately the same scale are the spectra of five elliptical galaxies. Each galaxy's spectrum is flanked above and below by a comparison spectrum of bright lines for the purpose of establishing the wavelength scale. The seven labeled comparison lines have the following wavelengths in angstroms:

a	3888.7	e	4471.5
b	3964.7	f	4713.1
c	4026.2	g	5015.7
d	4143.8		

Note the two dark notches in each galaxy's spectrum; from left to right, they are the K and H absorption lines of ionized calcium. Due to the expansion of the universe, the galaxies' spectra (including the calcium lines) are shifted redward to longer wavelengths. The displacement of the K and H lines in each case is indicated approximately by the length of the horizontal arrow.

1. *Scale of spectra.* In a comparison spectrum, measure the distance in millimeters between two widely spaced lines, estimating to tenths of a millimeter if possible. Find the difference in wavelength between the same lines from the above list, and divide by the measured distance to obtain the spectrum's scale in angstroms per millimeter. Because these are grating spectra, the scale should not depend on which pair of lines is selected.

Do this for several comparison spectra, and average the results.

2. *Observed wavelengths.* For each galaxy, measure the distances in millimeters (and tenths, if you can) from the redshifted K and H lines to the same identified comparison line. Multiply every distance by the average scale value just determined, to obtain $\Delta\lambda$, the wavelength difference in angstroms. Add $\Delta\lambda$ to, or subtract it from, the comparison-line wavelength (depending on whether the galaxy line is to the right or left of the comparison line) to find the observed wavelength, λ', of the redshifted galaxy line.

Use the table above, or a copy of it, to keep an orderly record of your measurements and calculated results.

3. *Velocities of Recession.* If the galaxies were not receding, the K and H lines would be at their *rest wavelengths,* λ, of 3933.7 and 3968.5 angstroms, respectively. The recessional velocity V corresponding to the redshifted wavelength, λ', is given by:

$$V = c\,(\lambda' - \lambda)/\lambda,$$

where c is the velocity of light (use 300,000 kilometers per second).

Calculate V for each of the five galaxies, doing this for both the K and H lines and averaging the result. Strictly speaking, V should be corrected for the sun's motion around the center of the Milky Way galaxy, but this correction is small and may be ignored here.

4. *Galaxy distances.* The photographs of the five galaxies are all to the same scale. First, measure each galaxy's diameter in millimeters and, if possible, tenths. For a noncircular image, measure the longest and shortest diameters and average.

The scale of the photographs is given by the bar at bottom, which is equal to 150 seconds of arc. Use it to convert each diameter from millimeters to seconds.

Next, convert the angular diameters to radians by dividing them by 210,000 (a suitable approximation to the number of seconds in a radian, 206,265). Since we have adopted 0.03 megaparsec as the linear diameter of these galaxies, the distance D in megaparsecs is readily calculated from

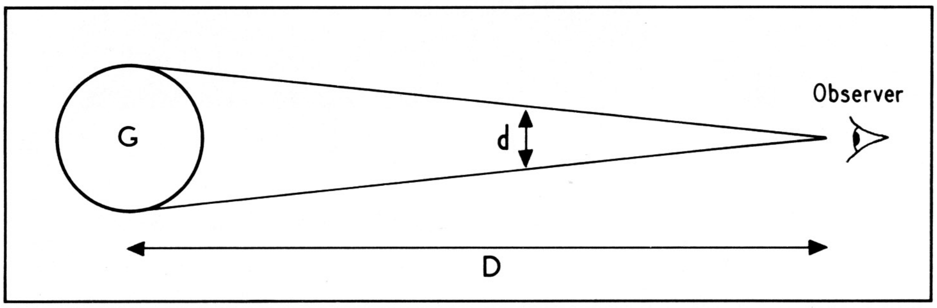
$$D = 0.03/d,$$

where d is the diameter in radians.

List your values of V and D for each galaxy in the table at right.

5. *Hubble diagram.* On a sheet of graph paper, plot the five galaxies, using distance as the horizontal scale and recessional velocity as the vertical. Draw the straight line that best represents the five points and also passes through the origin.

The Hubble constant, H, is this line's slope, obtained by dividing any recession



The author's diagram shows the relation between the distance, *D*, of a galaxy and the angular diameter, *d*, which it subtends to an observer.

DISTANCE DETERMINATIONS

Galaxy in	Diameter on photograph (mm.)	Diameter in seconds	Diameter in radians (d)	Distance in megaparsecs (D)
Virgo				
Ursa Major				
Corona Borealis				
Bootes				
Hydra				

velocity on the line by the corresponding distance. What value of H do you get, and how does it compare with the generally accepted value?

6. *Age and size of the universe.* Hubble's constant is related to the "age" and "radius" of the universe, in a manner which depends on the choice of a cosmological model. For a simple-minded estimate of the "radius" of the universe, calculate D from the formula $V = H \times D$ when the velocity of light c is substituted for V. To convert this radius from megaparsecs into light-years, multiply it by 3.3×10^6.

In most cosmological models, the "age" of the universe is of the order of $1/H$, but cannot exceed it. First, divide your value of H by 10^{12} to convert it from units of kilometers per second per megaparsec to units of kilometers per year per kilometer. Then the reciprocal of this transformed value is a rough estimate of the "age" of the universe in years.

FINAL RESULTS

Galaxy in	Average recession velocity (km./sec.)	Distance (Mpc.)
Virgo		
Ursa Major		
Corona Borealis		
Bootes		
Hydra		

How does this age compare with the known age of the earth? the sun? the oldest stars?

NOTES FOR TEACHERS

Students should have little difficulty in completing the bulk of this exercise in two hours. If necessary, it can be shortened by asking students to determine the redshifted wavelength of that part of the continuum which lies midway between the K and H lines, rather than the K and H lines separately. If so, use 3951.1 angstroms as the value for the rest wavelength.

While it is to be hoped that no more major revisions of Hubble's constant are in store, any future change can be allowed for by changing the linear diameter, 0.03, to $0.03 \times 50/H'$, where H' is the new value of Hubble's constant.

Some additional questions that students might wish to consider are: 1. What effect has the progressive downward revision of H from Hubble's original value to the present estimate had on our conception of the time and distance scales of the universe? 2. Is Hubble's "constant" really constant? What would happen if it increases or decreases with distance? 3. What happens to galaxies whose distance is equal to the "radius" of the universe? 4. Is the simple Doppler formula that was used valid for the velocities obtained? 5. Is our assumption of inverse proportionality between size and distance valid in a non-Euclidean universe?