The 762 nm emissions of sprites

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We report the 762 nm emissions in sprites recorded by the ISUAL experiment onboard the FORMOSAT-2 satellite. The 762 nm imager filter is centered at 763.3 nm with a 7 nm bandwidth at 50% transmittance. Sprite emissions in this passband include the N₂ first positive (1PN₂) bands, (2, 0) and (3, 1), the O₂ atmospheric (atm) band (0, 0), and the hydroxyl (4, 0) emissions. Because these mixed emissions cannot be resolved in the 762 nm narrowband filter, a zero-dimensional plasma chemistry model is used to estimate the expected relative intensities of these emission bands in sprites. The computed 1PN₂ brightness in a single streamer is 1.4 MR and 2.6 kR for the O₂ atm band emissions at frame integration times of 30 ms. In the 762 nm passband, the 1PN₂ emissions are the dominant emissions in sprites, and the ratio of 1PN₂ to O₂ atmospheric emissions is ∼500, while the hydroxyl emissions can be neglected. In this ISUAL 762 nm campaign, the brightest sprite out of the four recorded events has possible O₂ atm band emissions that lasted more than 90 ms, and its observed brightness is consistent with the model prediction. Even though the lightning 762 nm emissions are strongly absorbed by O₂ below 60 km, the ISUAL observed parent lightning emissions in this passband are still more than a factor of two brighter than those from ISUAL observed sprites. Hence for spacecraft nadir TLE detection missions, 762 nm bands may not be used as the sole signature to identify sprites, and auxiliary emission bands are needed.


1. Introduction

[2] A sprite is one of several types of transient luminous events (TLEs) induced in the upper atmosphere by thunderstorms, and it usually occurs in the altitude range of 40–90 km [Sentman et al., 1995]. Telescopic images of sprites show the existence of narrow filamentary plasma structures in sprites, called streamers. The radius of sprite streamers is tens to hundreds of meters at sprite altitudes of ~70 km [Gerken et al., 2000; Gerken and Inan, 2003]. The measured streamer radius is consistent with predictions from numerical simulation of streamers [Pasko et al., 1998]. A streamer can initiate from a single electron avalanche at sprite altitudes and develop into a propagating ionization wave driven by a weak background electric field [Liu and Pasko, 2004, and references therein]. The inferred electric field derived from spectroscopic analysis [Kuo et al., 2005] or streamer modeling [Liu and Pasko, 2005; Liu et al., 2006] indicates a high electric field in the streamer head, which can trigger a series of chemical reactions. Sentman et al. [2008] developed a plasma chemical streamer model (hereinafter referred to as S08) to quantify the local chemical effects for a passing sprite streamer at 70 km altitude.

[3] In this paper, we utilize the S08 model with an updated chemical set [Kamaratos, 2009], which is discussed in section 3.2. The modified S08 modeling results are compared to the inferred 762 nm sprite spectroscopic data recorded by the ISUAL experiment. The main purpose of this study is to understand the spectroscopic content of the ISUAL sprites taken through a narrowband 762 nm filter, taking into account the various coupled physical processes producing optical emissions. It is expected that for spaceborne observations, the sprite emissions in the 762 nm range will primarily come from the sprite body due to strong absorption in the O₂ atmospheric band (O₂ atm) by O₂ below 60 km, as indicated by various ground-based measured sprite spectra [Mende et al., 1995; Green et al., 1996; Hampton et al., 1996; Kanmae et al., 2007, 2010]. The 762 nm filter for observing sprites has been proposed for several spacecraft projects, such
The passband of the ISUAL 762 nm imager filter and the emission bands that fall within this passband. The 1PN2 (2, 0) and (3, 1) bands, O2 atmospheric band (0, 0), of the excited electronic state emissions is the population of the upper state vibrational level in the band emissions, and band above the altitude of N-MendeMeinel emissions that fall vN emissions originate well above the quenching filter [Chou et al., 2008; Chang et al., 2010; Lee et al., 2010]. To study the possible existence of the O2 atm band and other emissions falling in the 762 nm passband, a special ISUAL 762 nm imager campaign was conducted from May to August 2008. In this campaign, six elves, two halos and four sprites were recorded by the 762 nm imager. The passband of the 762 nm filter is the black line.

as it was used in the narrowband spectroscopic cameras of the SPRITE-SAT project [Takahashi et al., 2010] and in the micro cameras and photometers of the upcoming TARANIS project [Blanc and Lefevre, 2006]. Here, we present spacecraft observations of 762 nm emissions in sprites. The results presented in this paper may be useful to improve the design of the 762 nm filters, in the analysis of sprite 762 nm emissions, and to anticipate possible difficulties that might be encountered in current and future spacecraft observations, especially for the nadir observation of sprites.

2. The ISUAL 762 nm Campaign
2.1. The Payload and the FORMOSAT-2 TLE Observations
[Chen et al., 2005]. Using the satellite information recorded with the TLE events, the altitude and location of the events can be determined by assuming that the bright center of the observed accompanying lightning–illuminated cloud has a height of 10 km [Kuo et al., 2008].
[6] After the ISUAL was launched on 21 May 2004 [Mende et al., 2005], during the first 5 year survey, more than 10,000 TLEs (sprites, halos, elves, and gigantic jets) were recorded by the ISUAL imager through a 1PN2 filter [Chen et al., 2008; Chou et al., 2010; Chang et al., 2010; Lee et al., 2010]. To study the possible existence of the O2 atm band and other emissions falling in the 762 nm passband, a special ISUAL 762 nm imager campaign was conducted from May to August 2008. In this campaign, six elves, two halos and four sprites were recorded by the 762 nm imager.

2.2. Sprite Emissions in the 762 nm Passband
[7] Theoretical spectra in Figure 1 indicate that the sprite emissions falling within the 762 nm passband include the first positive bands of molecular nitrogen (1PN2), the atmospheric band of molecular oxygen, and the hydroxyl (OH) emissions. Figure 1 shows the theoretical rotational–vibrational spectrum of 1PN2, which was computed based on the work of Kuo et al. [2008]. The assumed rotational temperature, 220 K, is the ambient temperature of the neutral species at 70 km altitude [Brasseur and Solomon, 1986].

[8] The intensity of a 1PN2 band for a given vibrational level $v$ is given by

$$I_{v',v''} = A_{v',v''} N_v,$$

(1)

where $N_v$ is the population of the upper state vibrational level $v$ in the band emissions, and $A_{v',v''}$ is the Einstein coefficient, which is the probability of radiative transition from the upper state vibrational level $v''$ to the lower state vibrational level $v'$ [Vallance-Jones, 1974]. In writing (1), it is assumed that the observed 1PN2 emissions originate well above the quenching altitude of 53 km [Vallance-Jones, 1974], so collisional quenching is not important. The population of the vibrational level $v$, $N_v$, is proportional to the Franck-Condon factor. The Franck-Condon factor is the transition probability from the zeroth vibrational level of the electronic ground state $N_2(X^2\Sigma^+)$ to vibrational level $v'$ of the excited electronic state $N_2(B^3Π_g)$, since the vibrational relaxation time is longer than the lifetime (7 μs) of the 1PN2 band above the altitude of 45 km [Kuo et al., 2008, and references therein]. The relative intensities of rotational levels for the $v'$ vibrational state of 1PN2 band follow the Boltzmann distribution, and the temperature of the Boltzmann distribution is assumed to be the neutral temperature [Laux, 1993].

[9] The percentage of the total 1PN2 emissions that fall in the ISUAL imager 762 nm passband is defined as the total 1PN2 emissions multiplied by the filter response curve. The response function of the 762 nm filter was preflight calibrated and the central wavelength is at 763.25 nm. Using the response curve, the percentage of the total 1PN2 emissions that fall in the ISUAL 762 nm passband is ~5%.

[10] The wavelength of the hydroxyl OH (4,0) emissions are computed using the coefficients reported by Meinel [1950] and Herman and Hornbeck [1953]. The population distribution of the vibrational levels for the OH upper state $X^3Π_{3/2}$ has characteristics of a Boltzmann distribution with...
The applied electric pulse field and the theoretical brightness of induced band emissions in a sprite streamer are shown. In our plasma chemical model: (a) the maximum applied electric field at streamer tip is $5\ E_k$, with $E_k$ being the conventional breakdown electric field, and (b) the volume emission rate (left) and the corresponding brightness (right) for the major band emissions.

$T_e$ of ~10,000 K [Khomich et al., 2008, p. 129, and references therein]. The relative population for the vibrational level ($v'=4$) is $N_{v'=4} \sim 0.06$. The ratio of the Einstein coefficient $A_{4,0}$ to the sum of $A_{4,0} + A_{4,-4}$ is ~0.002 (0.13/69.38) [Goldman et al., 1998]. Therefore, the ratio of the OH (4 0) to the total hydroxyl emissions is ~0.0001, which is comparable to the result of ~0.0002 derived from the observation data listed by Khomich et al. [2008, Table 2.14]. The percentage of the total OH (4,0) emissions that fall in the passband of the ISUAL 762 nm filter is 28%. Hence, the percentage of the total OH emissions in the 762 nm filter is ~0.003% (0.0001 × 28%), which is small enough to be neglected.

[11] The intensity of the $O_2$ atm (0,0) rotational spectrum is computed using the information provided by Babcock and Herzberg [1948] and is shown as the green curve in Figure 1. The intensity ratio of $O_2$ atm (0,0) to the total $O_2$ atm emission band is 88.5% [Khomich et al., 2008, Table 2.27]. The percentage of the total $O_2$ atm (0,0) emissions that fall in the ISUAL 762 nm passband is ~84.6%. The expected percentage of the detectable $O_2$ atm (0,0) from the total $O_2$ atm band emissions is therefore ~75% (88.5% × 84.6%).

2.3. Absorption of $O_2(b^1\Sigma^+_u)\rightarrow O_2(X^3\Sigma^+_g)$

[12] The oxygen density profile between the altitudes of 0 km to 1000 km is obtained from the MSIS model [Hedin, 1991]. This density profile is useful in computing the column integrated molecular oxygen density, which is responsible for the absorption of 762 nm emissions by the oxygen along the raypaths that connect the ICCD imager and the TLEs. For the sprite event on 15 May 2008, 2157:30.446, the event location is determined to be ~7°N and 15°E. Since the absorption cross section of molecular nitrogen is very small [Jursa, 1985, p. 22-4], the molecular nitrogen absorption of the 762 nm emissions can be neglected. The column integrated molecular oxygen density is $2.1 \times 10^{21}$ molecules cm$^{-2}$, and the absorption cross sections at the peaks of the P (760.8 nm) and the R (763.8 nm) branches of the $O_2(b^1\Sigma_u^+)\rightarrow O_2(X^3\Sigma_g^+)$ emissions are $3.2 \times 10^{-24}$ and $2.2 \times 10^{-24}$ cm$^2$ [Greenblatt et al., 1990], respectively. Using the Lambert’s law, the atmospheric transmittance can be expressed as $e^{-L(h,d,\lambda)}$ where $L(h, d, \lambda)$ is the column integrated molecular oxygen density as a function of altitude ($h$), distance ($d$), and wavelength ($\lambda$); the laboratory-measured cross section $\sigma(\lambda)$ is in units of cm$^2$. The transmittances for sprite emissions at wavelengths of 760.8 and 763.8 nm from an altitude of 70 km and at a distance of 2800 km are 99.3% and 99.5%, respectively. Therefore, the absorption percentage of the 762 nm emissions from the sprite on 15 May 2008, 2157:30.446 at 70 km altitude is less than 1%.

3. The Plasma Chemistry Model for a Sprite Streamer

3.1. The Zero-Dimensional Plasma Chemistry Model

[13] A zero-dimensional plasma chemistry model was first developed by Sentman et al. [2008] to study the localized chemical reactions induced by the electric discharges associated with sprite streamers over a wide range of timescales, and the associated radiative emissions. The timescales span several orders of magnitude from μs to tens of seconds. Taking the ionization pulse depicted in Figure 2a as an example, the pulse length is assumed to be 100 m and the propagation speed is ~10$^7$ m s$^{-1}$ [McHarg et al., 2007; Stenbaek-Nielsen et al., 2007; Li and Cummer, 2009], hence a fixed point in the streamer head will experience ~10 μs of transient E field. Though the driving electric field only lasts for 10 μs, the induced plasma chemical reactions can continue for several seconds. Within the passband of the 762 nm filter,
the radiative lifetime of the 1PN₂ emissions (5.4 μs) is found to be about six orders of magnitude shorter than that for the O₂ atm band (~12 s). The applicable temporal window of the S08 model can span from microseconds to tens of seconds. Hence, the relative radiative intensities of the sprite streamer emissions, including 1PN₂, 1NN₂, O₂ atm and OH band emissions, can be traced over a wide range of timescales.

3.2. The Chemical Reaction Set in the Plasma Chemistry Model

[14] In this work, the basic chemical reaction set and species of S08 are adopted with the inclusion of additional modifications that were discussed by Kamaratos [2009]. For example, Kamaratos [2009] suggested that the following electron impact processes should be considered for the lower vibrational states of N₂(X¹Σ⁺, v = 1–4),

\[ e^- + N₂ \rightarrow e^- + N₂(X¹Σ⁺, v = 1–4), \]  

(2)

and for the higher vibrational states of N₂(X¹Σ⁺, v = 5–9),

 \[ e^- + N₂ \rightarrow e^- + N₂(X¹Σ⁺, v = 5–9). \]  

(3)

[15] The following vibrational kinetics is also considered [Kamaratos, 2009],

\[ E^4*: N₂(A¹Σ⁺) + N₂(X¹Σ⁺, v = 5–9) \rightarrow N₂(B³Π_u^+) \]

\[ + N₂(X¹Σ⁺, v' < v'), \]  

(4)

where the chemical reaction rate constant is \( 4 \times 10^{-11} \text{ cm}^3 \text{ molecules}^{-1} \text{ s}^{-1} \) [Piper, 1992, 1994]. Also, the following reaction is added as discussed in [Kamaratos, 2009],

\[ E^5*: N₂(a¹Σ_u^-) + O₂ \rightarrow N₂(B³Π_u^+) + O₂, \]  

(5)

[16] The rate of this chemical reaction constant is \( 3.2 \times 10^{-11} \text{ cm}^3 \text{ molecules}^{-1} \text{ s}^{-1} \) [Umemoto et al., 2003]. In addition, the reaction rate constant of the following reaction is changed:

\[ N₂(a¹Σ_u^-) + N₂ \rightarrow N₂(B³Π_u^+) + N₂, \]  

(6)

to be 0.02 times the value given in S08 model [Kamaratos, 2009], thus is \( 4 \times 10^{-15} \text{ cm}^3 \text{ molecules}^{-1} \text{ s}^{-1} \). Finally, for species N₂(A²Π_u^+) and N₂(B²Σ_u^+) and the processes

\[ e^- + N₂ \rightarrow 2e^- + N₂^2 (A²Π_u) \]  

(7)

and

\[ e^- + N₂ \rightarrow 2e^- + N₂^2 (B²Σ_u^+), \]  

(8)

the branching ratios are specified to be 0.535 and 0.145 times of the N₂ ionization (\( e^- + N₂ \rightarrow 2e^- + N₂^2 \)) by Van Zyl and Pendleton [1995], respectively. Therefore, the reaction rate constants for reactions (7) and (8) are 0.535 and 0.145 times that of the N₂ ionization rate (\( e^- + N₂ \rightarrow 2e^- + N₂^2 \)). The electron impact ionization processes, reactions (7) and (8), are used for estimating the emission intensity of the N₂ Meinel band and the N₂ first negative band (1NN₂) emissions. For the O₂ atm band emissions, a two-step Barlt-type process, a three-body recombination reaction, [McDade et al., 1986; Marsh et al., 2002; Belyaev et al., 2006] is considered,

\[ O(³P) + O(³P) + M \rightarrow O₂(b¹Σ_u^+) + M, \]  

(9)

where M either is molecular nitrogen or oxygen. The rate constant for (9) is \( \sim \varepsilon \times 4.7 \times 10^{-19}(300/T)^2 \) where T is the temperature (in Kelvin) and the coefficient \( \varepsilon \sim 0.1 \) since the number ratio of O₂(b¹Σ_u^+) to all the O₂ substates from the total recombination reaction [McDade et al., 1986] is \( \sim 0.1 \). For OH emissions, the relevant chemical reactions are included in the S08 model [Sentman et al., 2008].

4. Model Results and the Observed Sprite 762 nm Emissions

4.1. Major Emissions in a Sprite Streamer

[17] Figure 2a shows the magnitude and duration of an impulsive E field moving along the symmetrical axis of a streamer from the streamer head moving toward the streamer body. The E field within the streamer is parameterized as the following:

\[ E(t) = E₀e^{−(t−t₀)/Δt²} , \]  

(10a)

and the streamer body (\( t > 18 \mu s \)),

\[ E(t) = E₀ \cdot t₀ + 3Δt ≤ t ≤ 1 ms, \]  

(10b)

where a high electric field at the streamer head lasts for \( Δt = 6 \mu s \) and a small E field in streamer body is assumed to persist for \( \sim 10 \) ms. These times are estimated values from assuming that the length of a streamer is \( \sim 10 \) km and the ionization wave at the streamer head moves at a speed of \( 10^7 \) m/s [McHarg et al., 2007; Stenbaek-Nielsen et al., 2007; Li and Cummer, 2009]. The peak magnitudes of the E field are \( E₀ = 5 E₂ \) at the streamer head and \( E₀ = 0.5 E₂ \) in streamer body, respectively. Here, \( E₂ \) is the conventional breakdown E field and is equal to \( \sim 220 \) V/m at altitude 70 km where the air density is \( 1.5 \times 10^{15} \) molecules \( \text{cm}^{-3} \). The adopted \( E₀ \) and \( E₂ \) values are similar to those used in the modeling of positive streamer propagating in the background E field \( \sim E₂ \) [Liu et al., 2009a, 2009b] and in the plasma chemical streamer model reported by Sentman and Stenbaek-Nielsen [2009].

[18] The computed major optical emissions in a streamer are shown in Figure 2b, and the density evolution of the major chemical species in a single streamer is shown in Figure 3. The dominant emissions (1PN₂, 2PN₂, 1NN₂ and N₂ Meinel) in a streamer are primarily from the electron impact excitations within the streamer head region. The streamer radius is assumed to be \( 25 \) m [Sentman et al., 2008], the computed peak photon emission rate of 1PN₂ is \( 8.4 \times 10^{11} \) photons \( \text{cm}^{-2} \text{ s}^{-1} \) and the corresponding peak 1PN₂ brightness is \( 2.5 \times 10^7 \times 8.4 \times 10^{11} \sim 2 \times 10^9 \) Rayleigh (2 GR) where 1 Rayleigh = \( 10^{10} \) photons \( \text{cm}^{-2} \text{ s}^{-1} \) in a 1 cm² column. The 2PN₂, 1NN₂ and N₂ Meinel emissions are 1.8 GR, 0.06 GR and 0.044 GR at the streamer head, respectively. The brightness of the streamer body is nearly two orders of magnitude lower than the brightness of the streamer head.
The dominant chemical species along the symmetrical axis of a single streamer: (a) the applied electric pulse field and (b) the number density evolution.

The brightness of 1PN2, 2PN2, 1NN2, and N2 Meinel emissions are estimated to be 24 MR, 1.8 MR, 2.1 kR and 0.6 kR in the streamer body, respectively.

[10] Even with a relatively low E field at the streamer body ($E_b = 0.5 E_k$), the radiative contribution from the electron impact excitations in the streamer body is only two orders of magnitude less than that from the emissions at the streamer head. After the removal of E field ($E_b = 0$), the 1PN2, 2PN2, and 1NN2 emissions decay immediately. The induced emissions, including O2 atmo, O(1D) 630 nm, O(1S) 557.7 nm and OH emissions, are produced by species lasting for more than 1 ms. The maximum brightness for the O2 atm emissions is 2.6 kR at 2 ms, O(1D) 630 nm is 30 R at 20 μs, O(1S) 557.7 nm is 6.5 kR at 1 ms, and OH emissions is 1.5 R after 1 s. One reason for the long lifetime of these emissions compared to those for electron impact excited emissions is the smallness of their radiative Einstein coefficients. Another reason is that the chemical reaction rates for producing the minor species are slow. Table 1 shows the transition, the wavelength, and the radiative Einstein coefficients for the major band emissions in a model sprite streamer.

4.2. The Chemical Reactions Leading to the O2 Atmospheric Band Emissions

[20] The possible pathways to generate the observed 762 nm emissions are discussed here, and the results are depicted in Figure 4. Figure 4a depicts the profile of the electric field in a sprite streamer. Figure 4b presents the number density of the O2 ($b^1\Sigma_g^+$) substate, which is the upper state of the O2 atm band. Figure 4c shows the temporal evolution of the related chemical reactions that lead to the production and the loss of O2 ($b^1\Sigma_g^+$), which are indicated by

<table>
<thead>
<tr>
<th>Table 1. Major Band Emissions of Sprite Streamers, the Upper and Lower States, the Emission Wavelengths, the Einstein Coefficients ($A_e$), and the Quenching Constants ($K_q$) for Molecular Nitrogen and Oxygen</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emission Band System</td>
</tr>
<tr>
<td>----------------------</td>
</tr>
<tr>
<td>1PN2</td>
</tr>
<tr>
<td>2PN2</td>
</tr>
<tr>
<td>1NN2</td>
</tr>
<tr>
<td>O2 atmosphere</td>
</tr>
<tr>
<td>O</td>
</tr>
<tr>
<td>O</td>
</tr>
<tr>
<td>OH</td>
</tr>
</tbody>
</table>

4Based on a private discussion [Liu and Pasko, 2005], the middle value of Einstein coefficient for major vibrational levels of upper state of band system [Gilmore et al., 1992, Table 19] is used in our model.

5Piper [1992].

6The lower Einstein coefficient [Vallance-Jones, 1974, p. 119] provides a better fit for our observation and is 74% of that listed by Gilmore et al. [1992, Table 19].

7With these values, the quenching height for 2PN2 and 1NN2 are ~30 and ~48 km [Pancheshnyi et al., 1997].

8Vallance-Jones [1974].

9Gilmore et al., 1992, Table 19.

10Piper [1992].
Figure 4. The chemical reactions leading to the O$_2$ atmospheric band emissions. (a) The applied electric field; the maximum electric field at the streamer tip is 5 E$_k$. (b) The accumulated density of the O$_2$(b$^1\Sigma_g^+$) upper state of the O$_2$ atmospheric bands. (c) The chemical reaction rate for the production and the loss of the O$_2$(b$^1\Sigma_g^+$) upper state. The reaction labels are the same as given in Appendix A of Sentman et al. [2008].

Table 2. Intensity of Sprite Streamer Emissions From This Work, the Observation Reported by Stenbaek-Nielsen et al. [2007], and the Streamer Model$^a$

<table>
<thead>
<tr>
<th>Altitude</th>
<th>Sprite Observation$^b$</th>
<th>This Model (E$_a$ = 5 E$_k$, E$_b$ = 0.5 E$_k$)</th>
<th>Streamer Model$^c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>75 km</td>
<td>Brightness (Rayleigh)</td>
<td>$4 \times 10^6$ to $3 \times 10^{11}$</td>
<td>$2.5 \times 10^8$</td>
</tr>
<tr>
<td>70 km</td>
<td>Photon flux (photons cm$^{-3}$ s$^{-1}$)</td>
<td>$3 \times 10^9$ to $&gt;2 \times 10^8$</td>
<td>$10^7$ (weak field) to $5 \times 10^8$ (strong field)</td>
</tr>
</tbody>
</table>

$^a$The modeled and the observed emissions are from the streamer tips, and the modeled photon fluxes are computed using the event distances reported by Stenbaek-Nielsen et al. [2007].

$^b$Stenbaek-Nielsen et al. [2007].

$^c$Liu and Pasko [2004, 2005].
Table 3. The Emission Ratios of 2PN/1PN and 2PN/1NN in Sprites From the ISUAL Observed Events, This Model, and the Streamer Model Reported by Liu et al. [2006, 2009a, 2009b]

<table>
<thead>
<tr>
<th>ISUAL Recorded Sprites</th>
<th>This Model; in Streamer Head Region</th>
<th>Streamer Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>2PN/1PN</td>
<td>0.24 ± 0.04</td>
<td>0.47</td>
</tr>
<tr>
<td>2PN/1NN</td>
<td>32.2 ± 11.2</td>
<td>36.3 (Eb ~ 3 Eb)</td>
</tr>
</tbody>
</table>

*It should be noted that the observed ratios are computed using the time-integrated emissions from all regions of sprites, while the model values are from a single sprite streamer.

1.5 × 10^{21} photons s^{-1}. Therefore, the estimated brightness is 10^{11} photons cm^{-2} s^{-1} × 25 m = 2.5 × 10^{14} photons cm^{-2} s^{-1} = 2.5 × 10^{15} Rayleigh. The brightness reported in the streamer modeling [Liu and Pasko, 2004, 2005; Liu et al., 2009a, 2009b] is 10^9 Rayleigh for a weak electric field case and 5 × 10^{10} Rayleigh for a strong electric field case.

Assuming a constant emission rate for a streamer head with a propagation velocity 10^7 m s^{-1}, the spatially integrated photons during 50 μs is ∼25 m × 25 m × (10^7 m s^{-1} × 50 μs) × 10^{11} photons cm^{-2} s^{-1} = 3.1 × 10^{22} photons s^{-1}. Assuming isotropic emissions and an estimated sprite distance of 335 km as reported by Stenbaek-Nielsen et al. [2007], the photon flux is 3.1 × 10^{22} photons s^{-1}4/π(3.35 × 10^8 cm)^2, or 2.2 × 10^{26} photons cm^2 s^{-1}. These model-derived parameters are all consistent with the observed values in sprite streams [Stenbaek-Nielsen et al., 2007; Stenbaek-Nielsen and McHarg, 2008].

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Table 4 summarizes the model brightness for the emissions 1PN, O_2 atm and OH in the passband (759.4–767.1 nm) of the 762 nm filter. The model brightness for these emission bands at the 30 ms integration time is listed in the second column of the Table 4, after correction for the detection percentage through the 762 nm passband (see discussions in section 2.4). The estimated 1PN, O_2 atm brightness in the 762 nm filter are 70 kR and 2 kR, respectively. The brightness ratio of 1PN to O_2 atm is ∼3.5. Therefore, it is generally difficult to distinguish the weaker O_2 atm (0, 0) emissions from the dominant 1PN band emissions. However, the luminous duration of the O_2 atm band can be up to 1 s, which is much longer than that for the 1PN. Therefore to resolve and confirm the existence of the 762 nm O_2 atm (0, 0) emissions, ISUAL recorded sprite emissions beyond the first frame (30 ms) have to be analyzed in detail.

4.4. The Intensity Ratio of the Sprite 1PN and the O_2 Atmospheric Band Emissions

To infer the relative emission intensity of the O_2 atm and the 1PN in sprites, we use the model intensities to evaluate the brightness of the emission bands in the 762 nm filter. Figure 5 shows the averaged brightness of the major sprite emission bands for six temporal bins of 30 ms each. The temporal length and the bin size are chosen to match the frame integration time and the six-frame recording length of the ISUAL imager. The estimated 1PN, 2PN and 1NN_2 brightness are 1.4 MR, 0.3 MR and 6.7 kR, respectively, for the first frame. As shown in Figure 2, the luminous duration (∼1 ms) of these band emissions is substantially shorter than the integration time (30 ms) of a single imager frame; see Figure 6. The model results indicate that the maximum brightness of the O_2 atm band, O(1D) 630 nm, O(1S) 557.7 nm and OH emissions is ∼2.6 kR, ∼0.6 R, ∼3.6 kR and ∼1.3 R, respectively. Since the chemical reactions leading to the O_2 atm band emissions have a lifetime of ∼12 s as discussed in section 4.2, the O_2 atm band emissions last and remain nearly constant over six imager frames of 180 ms. The average brightness of the O_2 atm band emissions is ∼2.6 kR as can be inferred from Figure 5.

Figure 5. The model brightness of major emission bands for a streamer at altitude of 70 km and with an exposure time of 30 ms. The choice of the exposure time is to close match that of the ISUAL 29 ms imager frame.
The sprite is determined to be between 58 and 70 km. The sprite initiation time was ∼1.5 ms after the trigger from AP data.

From Figures 6a–6e, this 762 nm sprite seems to contain two types of emissions. The first type is the primary emissions, and they are so relatively short-lived that they only show up in the first two frames of Figure 6. The average brightness of this sprite in frames a and b is ∼173 kR, and the maximum brightness is ∼1 MR. The second type is weak continuous emissions that persist into frames c, d, and e, as may be discerned in Figure 6. The second group of emissions could have begun from the first frame and have persisted for more than 90 ms. The maximum brightness for the persisting emissions in frames c, d, and e are 28.6, 27.6, 23.3 kR, respectively. The recorded O₂ atm (0, 0) in the sprite is estimated to be ∼30 kR, while its 1PN₂ brightness is ∼1 MR. Hence for this sprite, the brightness of this persisting emissions is about 1/33 that of the 1PN₂ in the 762 nm passband, which is consistent with the model predicted value of 1/35 (see section 4.4). The only significant emissions that could exist in the temporal range is the O₂ atm (0, 0) emissions.

Table 4. The 1PN₂, O₂ Atmospheric Band (0, 0) and OH Band (4, 0) Emissions From a Sprite Streamer That Fall Within the ISUAL 762 nm Filter Passband

<table>
<thead>
<tr>
<th>Emission Type</th>
<th>Modeled Brightness</th>
<th>Detected Percentage</th>
<th>Estimated Brightness</th>
<th>Brightness Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1PN₂</td>
<td>1.4 MR</td>
<td>~5%</td>
<td>70 kR</td>
<td>1</td>
</tr>
<tr>
<td>O₂ atm (0, 0)</td>
<td>2.6 kR</td>
<td>~75%</td>
<td>2 kR</td>
<td>~0.03</td>
</tr>
<tr>
<td>OH (4, 0)</td>
<td>1.3 R</td>
<td>0.003%</td>
<td>0.004 R</td>
<td>~7 × 10⁻⁸</td>
</tr>
</tbody>
</table>

*The columns present the theoretical emission band brightness from our plasma chemistry model, the percentage of the band emissions that fall within the ISUAL 762 nm filter, the predicted brightness in the 762 nm passband, and the estimated brightness ratio if a single streamer can correctly reflect the ratio of the whole sprite.
whose parent chemical reaction has a lifetime of ~12 s. Hence it can be concluded that the persisting 762 nm emissions in this sprite is from the O$_2$ atm (0, 0) band.

[30] Among the four sprites recorded by the ISUAL imager through the 762 nm filter, only the brightest event analyzed above has discernible O$_2$ atm (0, 0) emissions. The other three sprite events either are too dim, or were recorded in the penultimate imager frame and the O$_2$ atm (0, 0) emissions are swarmed by the dominant 1PN$_2$ emissions, thus they cannot be resolved.

5. Discussion

[31] The minor differences between the plasma chemical model employed here and that by Sentman et al. [2008] lie in the slightly different sets of chemical equations used in the respective works. In the present work, the reactions (2)–(6) associated with the excited species N$_2$(a$^1$Σ$_u^+$) and the N$_2$ vibrational kinetics were included, as suggested by Kamaratos [2009] and references therein.

[32] The dominant process in producing N$_2$(B$^2$Π) is the cascading process N$_2$(C$^2$Π$_u$) → N$_2$(B$^2$Π) + hν (2PN$_2$). The radiative life time of this cascading process is very short, 50 ns, which is the inverse of the corresponding Einstein coefficient listed in Table 1. However, after the production of N$_2$(a$^1$Σ$_u^+$), the quenching collisions with O$_2$ or N$_2$ can cross N$_2$(a$^1$Σ$_u^+$) into N$_2$(B$^2$Π) and cause an enhancement of the 1PN$_2$ emissions [Kamaratos, 2009, and references therein]. In our model single streamer, the maximum density of the excited species N$_2$(a$^1$Σ$_u^+$) can be up to $3.8 \times 10^6$ cm$^{-3}$, ~67% of the maximum density ($5.6 \times 10^6$ cm$^{-3}$) of N$_2$(B$^2$Π)$_g$; Figure 3.

[33] That affect the excited species N$_2$(a$^1$Σ$_u^+$) are the radiative transition (Ogawa-Tanaka-Wilkinson-Mulliken band system, a$^1$Σ$_u^+$→X$^1$Σ$_g^+$) [Lofthius and Kruipenie, 1977], the production of two O atoms [Kossyi et al., 2002], the quenching processes with N$_2$ or O$_2$ [Umamoto et al., 2003], and the reactions that produce N, O, and N$_2$ [Sentman et al., 2008, and references therein]. The radiative lifetime of a$^1$Σ$_u^+$→X$^1$Σ$_g^+$ is 0.5 s, since its Einstein coefficient is 2 s$^{-1}$. Comparing with Einstein coefficients of other N$_2$ emission bands (Table 1), the Einstein coefficient of the radiative transition a$^1$Σ$_u^+$→X$^1$Σ$_g^+$ is smaller. The radiative transfer calculations of this transition is relatively weak. Besides, the 108–200 nm emissions in the Ogawa-Tanaka-Wilkinson-Mulliken band system [Lofthius and Kruipenie, 1977] is not considered of interest in the study of 762 nm emissions of sprites.

[34] The quenching processes with O$_2$ and N$_2$ are N$_2$(a$^1$Σ$_u^+$) + O$_2$ → N$_2$(B$^2$Π)$_g$ + O$_2$ and N$_2$(a$^1$Σ$_u^+$) + N$_2$ → N$_2$(B$^2$Π)$_g$ + N$_2$. The quenching reaction rate constant is four orders of magnitude higher for O$_2$ than for that of N$_2$ [Kamaratos, 2009, and references therein]. The fractional reduction rate $\left(\frac{[N_2(a^1\Sigma_u^+)]}{[N_2(a^1\Sigma_u^+)]}\right)$ of N$_2$(a$^1$Σ$_u^+$) through the quenching reaction with O$_2$ is the O$_2$ density ($3 \times 10^{14}$ cm$^{-3}$) times the reaction rate constant $(3.2 \times 10^{-11}$ cm$^3$ s$^{-1}$) = 10$^5$ s$^{-1}$. The time for the density of N$_2$(a$^1$Σ$_u^+$) to drop from 10$^5$ cm$^{-3}$ to 1 cm$^{-3}$ is $\ln(10^5)$ / 0.1 ms ~ 1 ms. The substate N$_2$(B$^2$Π)$_g$ has a maximum density of 10$^5$ molecules cm$^{-3}$. Hence, the photon emission rate is estimated to have a maximum value of $10^8$ cm$^{-3} \times 1.7 \times 10^4$ s$^{-1}$ ~ $10^9$ photons cm$^{-3}$ s$^{-1}$.

[35] With the inclusion of these new vibrational kinetics (section 4.1), the density of N$_2$(X, v = 4) and N$_2$(X, v > 4) are computed to be increased to $2.3 \times 10^5$ and $1.2 \times 10^5$ molecules cm$^{-3}$, respectively, in ~10 µs, under the driving of a transient electric pulse that is $E_k$ at the streamer tip and 0.5 $E_k$ in the streamer body during a ~1 ms interval. The N$_2$ density is ~$1.2 \times 10^5$ molecules cm$^{-3}$ at the altitude of 70 km. The ratio of N$_2$(X, v > 4) to N$_2$ is only $4 \times 10^{-7}$, two orders of magnitude lower than the assumed $10^{-3}$ by Kamaratos [2009]. Hence, the production rate of N$_2$(B$^2$Π)$_g$ in the 1 ms E field driving period is expected to be weaker than that by Kamaratos [2009]. The density of N$_2$(A$^3\Sigma_u^+$) is up to $1.4 \times 10^5$ molecules cm$^{-3}$, and rate of the chemical reaction in equation (4) is $6.7 \times 10^4$ molecules cm$^{-3}$ s$^{-1}$ ($4.0 \times 10^{11}$ cm$^{-3}$ molecule$^{-1}$ s$^{-1}$ $\times 1.4 \times 10^5$ molecules cm$^{-3}$ $\times 1.2 \times 10^5$ molecules cm$^{-3}$). That is also two orders of magnitude lower than the expected reaction rate for the production of N$_2$(B$^2$Π)$_g$ in reaction (4). The change rate of N$_2$(B$^2$Π)$_g$, $\partial[N_2(B^2\Pi)]/\partial t$, is $[N_2(A^3\Sigma_u^+)] \times [N_2(X, v > 4)] \times$ reaction rate = $1.4 \times 10^5$ molecules cm$^{-3}$ $\times 1.2 \times 10^5$ molecules cm$^{-3}$ $\times 4 \times 10^{-11}$ cm$^{-3}$ molecule$^{-1}$ s$^{-1}$ = $10^9$ molecules cm$^{-3}$ s$^{-1}$. Therefore, 1 ms after switching off the electric field, the density of N$_2$(B$^2$Π)$_g$ drops down to $10^5$ molecules cm$^{-3}$ $\sim$ 100 molecules cm$^{-3}$. The photon emission rate is thereby reduced from the maximum value to $10^5$ molecules cm$^{-3}$ $\times 1.7 \times 10^4$ s$^{-1}$ ~ $10^9$ photons cm$^{-3}$ s$^{-1}$.

[36] To track the temporal evolution of the N$_2$(B$^2$Π)$_g$ density after switching off the driving E field, four types of reactions are considered, and the production and the loss rates of N$_2$(B$^2$Π)$_g$ via aforementioned reactions: (1) the cascading 2PN$_2$ radiative reaction, R105 (50 ns lifetime), (2) the quenching processes of N$_2$(a$^1$Σ$_u^+$) with O$_2$ and N$_2$ (~1 ms persisting duration), R100 and E5*, where reaction E5* indicates N$_2$(a$^1$Σ$_u^+$) + N$_2$ → N$_2$(B$^2$Π)$_g$ + N$_2$, listed before, (3) chemical excitation occurring by way of the energy pooling of N$_2$(A$^3\Sigma_u^+$) or O$_2$(a$^1$Δg), R91 and R90, and (4) the N$_2$ vibrational kinetics that can last for several milliseconds, E4*, are shown in Figure 7.

[37] In Figure 3, the density profile of N$_2$(a$^1$Σ$_u^+$) denoted by the black curve is tightly correlated with that of N$_2$(B$^2$Π)$_g$. This reflects the fact that the quenching cross process of N$_2$(a$^1$Σ$_u^+$) leading to increase of in N$_2$(B$^2$Π)$_g$ is effective for a period of ~1–2 ms. Comparing with the other processes, the vibrational excitation of N$_2$(B$^2$Π)$_g$, N$_2$(A) + N$_2$ (X, v = 5 − 9) → N$_2$(B) + N$_2$ (X, v′ < v), can persist for several minutes because of the long vibrational relaxation time of ~5–7 min [Picard et al., 1997]. Hence after switching off the E field, the production of N$_2$(B$^2$Π)$_g$ is mainly sustained by N$_2$(X, v = 5 − 9).

[38] As shown in Figure 7a, after about a couple of milliseconds from switching off the electric field (at 1 ms), the dominant N$_2$(B$^2$Π)$_g$ producing reactions in the afterglow region are the reactions R90 (green dotted lines) and E4* (green dashed lines). Comparing with the reaction R91, N$_2$(A) + N$_2$(A) → N$_2$(B) + N$_2$ (X, v′ < v), the reaction R90 N$_2$(A) + O$_2$(a) → N$_2$(B) + O$_2$, is expected to have a higher reaction rate than that of the R91 after switching off the E field; Figure 7a. The number density of O$_2$(a$^1$Δg) is expected to decrease slower than that for N$_2$(A$^3\Sigma_u^+$), shown in Figure 3.
and the hydroxyl (OH) emissions below 60 km, the sprite emissions in this band were observed to persist for 1 s, after the sprite initiation. With the luminous duration and the brightness consistent or close to that predicted, it is therefore concluded that the long persisting emissions detected in the 762 nm sprites was from the O₂ atm (0, 0) bands.

[42] In the ISUAL 762 nm sprite campaign, bright 762 nm lightning emissions were also detected; even though they are expected to be severely absorbed by molecular oxygen in the lower atmosphere. Through the 762 nm narrow band filter, the parent lightning was found to be nearly twice as bright as the sprite. Hence for spacecraft nadir TLE-detection missions, 762 nm bands may not be used as the sole signature in differentiating sprite events and additional auxiliary emission bands needed to be explored.

6. Summary

[41] To understand the origin of these emissions, a zero-dimensional plasma chemistry model was employed to compute the luminous duration and the brightness of the major sprite emissions in the ISUAL imager 762 nm passband. For a sprite streamer at 70 km, if the pulse electric field is 5 Eₖ in the streamer head (6 μs) and 0.5 Eₖ in the streamer body (1 ms), the model indicates the first positive bands of molecular nitrogen (1PN₂), the atmospheric bands of molecular oxygen (O₂ atm), and the hydroxyl (OH) emissions are the major sprite emissions in this passband. The theoretical brightness of the 1PN₂, O₂ atm, and OH emissions in a single streamer is 1.4 MR, 2.6 kR, and 1.3 R, respectively, averaging over 30 ms. After converting and factoring in the 762 nm filter transmittance of the 1PN₂, O₂ atm, and OH band emissions, the predicted brightness of these bands in the long-duration sprite is 1 MR, 30 kR, and ~0, respectively. The model further indicates that in the 762 nm passband only the O₂ atm emissions will persist for 1 s, after the sprite initiation. With the luminous duration and the brightness consistent or close to that predicted, it is therefore concluded that the long persisting emissions detected in the 762 nm sprites was from the O₂ atm (0, 0) bands.

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